CONTROLLED FLUID RECIRCULATION FOR EFFICIENT HYDRAULIC ACTUATION OF CONSERVATIVE LOADS

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List of Symbols

A	Piston area
B	Damping effect
Be	Bulk modulus
C	Valve Coefficient
$CC_i(U)$	Control constraint
g(x,u)	Function to be minimized
H(x,p,u)	Hamiltonian
$h(x(t_f))$	Final condition
$h(SC_i)$	Sign Function
i	Current input to electrohydraulic valve
J(x,u)	Performance measure
K	Valve sizing constant
K_i	Gain constant
k_1	Leakage coefficient
L	Piston stroke
M	Mass of load
p	Lagrange multiplier or costate
P_i	Pressure at i side
P_s	Supply pressure
P_r	Return pressure
P_v	Valve pressure drop
Q_a	Flow caused by actuator movement
Q_i	Control flow
Q_ℓ	Leakage flow
Q_c	Compressible flow
R	Weighting factor
$SC_i(X)$	State constrain
SGN	Sign function
t	Time
t_f	Final Time
U	Control vector
U*	Optimal control vector
U_m	Upper bound for control vector
V_0	Half of the total volume of actuator
X	State vector
X*	Optimal state vector
X1D,X2D,X3D,X4D	Desired states
y	Displacement

Introduction

Hydraulic actuators have traditionally found application in servo systems requiring high bandwidth, high power per unit mass, and permitting low overall energy efficiency, the presence of hydraulic oil, and a rather bulky power supply. This paper presents a theoretical, analytical and experimental study of controlled fluid recircultion around an actuator to provide a more energy efficient hydraulic servo system for motion control while retaining the traditional advantages.

The four way servovalve conventionally used in high performance hydraulic applications is a throttling device composed of four variable orifices arranged to form a Wheatstone bridge. (See Figure 1.) Spool valves and jet pipe valves are its common implemtations. The orifices are mechanically connected as shown by the dashed lines in Figure 1. Two of the orifices are open at any time, depending on the direction of fluid flow. The fluid supply is usually a pump, an accumulator, and a means of regulation to maintain the supply fluid at almost constant pressure. All flow to the actuator comes from the fluid supply through a valve orifice. All flow out of the actuator goes through another valve orifice and returns to a sump at much lower constant pressure, often atmospheric. The power required is dependent only on the pressures and the volumetric flow rate of fluid from the supply. Integrating this power over time gives the energy consumed which is equal to the supply pressure and fluid volume taken from the supply (proportional to the travel of the actuator, assuming negligible leakage). Notice that the power consumed has no direct relationship to the power required by the load. Much of the power is dissipated to heat across the valve orifices. Even when power is being taken out of the load, power must still be supplied from the pump and accumulator.

Power consumption may be reduced by keeping the supply pressure as low as possible. The minimum supply pressure permitted is determined by the maximum pressure required and perhaps by flow demands to be supplied from the accumulator. Further reduction of power consumption requires modification of the hydraulic circuit. A variable supply pressure would reduce power consumption during all phases of operation. Regenerative braking of the load would reduce the power required even further by returning energy taken from the load to the supply. Practical means for implementing these strategies that retain the high performance of the hydraulic servo do not exist. The variable jet pump valve [1] is a variable fluid transformer which potentially matches the supply and load impedances for better overall efficiency. The method explored here uses controlled fluid recirculation from the actuator, through a two way valve, and back to the actuator to reduce energy consumption.

Controlled fluid recirculation is appropriate for loads which conserve much of the energy provided to them. Examples are inertial loads, gravity loads, and spring loads. Inertial loads are considered exclusively throughout the remainder of this paper, although they represent a more demanding application of the strategy. These types of loads occur in robot arms, antenna, hoists, and machine tools. If the movement end point is known, the motion and the control of fluid recirculation can be optimized. This is the problem considered in detail below. If the motion end point is not known in detail or if the motion is completely specified at each point in time, controlled fluid recirculation is still potentially valuable, but this case is not considered below.

This paper describes the hydraulic circuit for fluid recirculation and the mathematical model of it used for analysis and controller design. The optimal control analysis of the circuit for moving an inertial load from one position to another in specified time with inequality constraints on states and control is then described. Implementation of a modification of the optimal control on a microcomputer and breadboard hydraulic system is presented with experimental results. All phases of the paper are oriented to producing an advanced, operating, motion control system with realistic component dynamics. Additional details are found in Punyapus [2].

A Circuit for Controlled Fluid Recirculation

The objective of energy saving by recirculation of fluid around the actuator can be achieved by several circuits. One alternative considered was four independently controlled orifices. This and other alternatives were rejected due to problems with cavitation or due to the expense of constructing the circuit. The circuit used consists of a 4-way valve and a two way valve as shown in Figure 2. It provides the following advantages when compared to the conventional four way valve circuit or alternatives.

1. Reduced energy consumption
2. Avoid pressure surge and cavitation
3. Stability enhancement through controlled leakage

The breadboard circuit was constructed from off-the-shelf components. The two way valve was constructed from a four way valve with two ports blocked.

Mathematical Model of the Circuit

To model the circuit, tests were performed to justify the assumptions regarding valve dynamics. With these assumptions and the fundamentals of conservation of mass and momentum, equations of motion were written. Simulatin using these equations were compared to experiment. The assumptions used were:

1. The orifice edges are assumed to be perfectly sharp. This is the characteristic of servovalve (MOOG 73-102) used.
2. The flow through each orifice is assumed to be simple orifice-type flow with negligible viscous effects, and the flow rate is assumed to change instantaneously with both pressure drop across orifice and orifice area., Time delay has been neglected so that a simple mathematical model will be obtained. The effects of neglecting time delay are expected when analytical and experimental studies are compared.
3. All of the connecting passes are short enough and wide enough to eliminate the effects of fluid mass on the flows through them.
4. Friction losses in the line and passes are included in the orifice losses, and leakage flow across the ram is assumed to be laminar.
5. Supply pressure is constant, and the bulk modulus of the fluid is taken to be constant within the range of pressure employed.
6. Exhaust pressure is assumed to be zero since it is very small compared to the operating pressure.

With these assumptions the conventional models for orifice flow, leakage flow, fluid compressibility, and conservation of mass and momentum can be used to obtain the equations summarized below. The schematic in Figure 3 and the list of symbols shows the nonmenclature for the equations.

Equations for Orifice Flow

$$Q_1 = Ci_1\sqrt{P_s - P_1}$$

$$Q_2 = Ci_2\sqrt{P_1 - P_2}, \qquad P_1>P_2$$

$$= Ci_2\sqrt{P_1 - P_2}, \qquad P_2>P_1$$

$$= SGN.Ci_2\sqrt{|P_1 - P_2|}$$

$$Q_3 = Ci_1\sqrt{P_2}$$

where $SGN. = 1.0, \quad P_1 > P_2$

$SGN. = -1.0, \quad P_2 > P_1$

Leakage Flow

$$Q_l = k_1(P_1 - P_2)$$

Compressible Flow

$$Q_{c1} = \frac{CV_1}{Be}\frac{dP_1}{dt}$$

$$Q_{c2} = \frac{CV_2}{Be}\frac{dP_2}{dt}$$

Conservation of Mass

For $P_1 > P_2$

$$CV_1 : \quad Q_1 - Q_\ell - Q_2 = Q_{c1} + Q_a$$

$$CV_2 : \quad Q_2 + Q_\ell - Q_3 = Q_{c2} - Q_a$$

For $P_2 > P_1$

$$CV_1 : \quad Q_1 + Q_\ell + Q_2 = Q_{c1} + Q_a$$

$$CV_2 : \quad -Q_2 - Q_\ell - Q_3 = Q_{c2} - Q_a$$

Conservation of Momentum

$$A(P_1 - P_2) = M\frac{d^2y}{dt^2}$$

The equations are then combined into four state equations in four state variables

Let $X_1 = y$. Then

$$X_2 = dy/dt$$

$$X_3 = P_1$$

$$X_4 = P_2$$

$$U_1 = i_1$$

$$U_2 = i_2$$

$$CV_1 = V_0 + AX_1$$

$$CV_2 = V_0 - AX_1.$$

Then state equations take the form:

$$\frac{dX_1}{dt} = X_2 \tag{1}$$

$$\frac{dX_2}{dt} = \frac{A}{M}(X_3 - X_4) \tag{2}$$

$$\frac{dX_3}{dt} = \frac{Be}{V_0 + AX_1}\left[CU_1\sqrt{P_s - X_3} - k_1(X_3 - X_4) - SGN.CU_2\sqrt{|X_3 - X_4|} - AX_2\right] \tag{3}$$

$$\frac{dX_4}{dt} = \frac{Be}{V_0 - AX_1}\left[k_1(X_3 - X_4) + SGN.CU_2\sqrt{|X_3 - X_4|} - CU_1\sqrt{X_4} + AX_2\right] \tag{4}$$

where $SGN. = SGN(X_3 - X_4)$

$SGN. = 0$ if $X_3 = X_4$

$SGN. = -1.$ if $X_3 < X_4$

$= +1.$ if $X_3 > X_4$

Estimation of Parameters

The model parameters were estimated on the basis of:

1. Direct measurements of system geometry.
2. Experimental measurements.
3. Data given by the manufacturer (MOOG Inc.).
4. Some parameters cannot be measured directly.
   These were estimated indirectly by finding values for them that made the simulation model behave like the experimental system.

Table 1 summarizes the parameters used in the model.

Optimal Control Analysis

Nonlinear time invariant system state equations with nonquadratic performance measures will lead to problems for which many basic approaches cannot be used. Linearization of the system state equations is prohibited by large variations of system states. The performance measure used combines the final state errors and the energy consumed. The weighting functions are applied to every term in the performance measure. The energy consumed is proportional to the flow in from the fluid supply. The flow in is linearly dependent on the control input U_1. This performance measure leads to a singular optimal control problem. For the problem to be well posed, constraints must be included in the problem. The inclusion of inequality constraints to a singular problem of a multivariable system will make the problem difficult if not impossible to solve analytically. A large computer with high speed must be used to solve the problem numerically off line. One of the indirect approaches generally used is the gradient method. This method applies the concepts of the calculus of variations to solve for the optimal path. To cope with the inequality and/or equality constraints, the method has to be modified. Then penalty function approach will be used to take care of the constraints in this paper.

In this particular application, the performance measure used is linear in control inputs. By using the variational approach, the local minimum is not guaranteed because the second derivative of the Hamiltonian is zero (instead of positive definite). Several trials for the initial guess of control inputs must be used. A global minimum is assumed if every trial converges to the same result.

The linear performance measure also leads to a singular problem when the first derivative of Hamiltonian is zero. Several references demonstrate the method of solving this kind of problem analytically. A singular problem usually leads to a bang-bang or bang-singular-bang control when the controls used are restricted by constraints. Simple problems of low order (< 3) linear systems have been solved successfully by many authors using Pontryagin's

Maximum Principle. For problems of higher order with nonlinear state equations, with constraints on controls and states, and with non-quadratic performance measures, analytical solutions are very difficult.

TABLE 1
List of Parameter and Units

P_s-supply pressure	1200.	psi
Be-bulk modulus	50,000.	psi
V_0-half cylinder volume	1.434	in^3
A-cylinder net area	.478	in^2
M-load mass	.259	$\frac{lb\text{-}sec^2}{in}$
k_1-leakage coefficient	.003	$\frac{in^3}{sec\text{-}psi}$
C-valve sizing constant	35.12	$\frac{in^4}{amp\text{-}sec \quad lb}$
U_m-maximum input	20.	milliamp

The gradient method or method of steepest descent consists of searching numerically for an optimum by making a new iteration move in the direction of the gradient, the direction of the maximum decrease (or increase) of the cost function. The method was developed and applied to the optimal control problem by Kelly [3] and Bryson and Denham [4]. Various modifications to include penalty functions, and other methods of treating equality and inequality constraints have been presented [5,6,7]. The gradient method has the characteristic that it converges very fast for a few iterations and tends to converge very slowly as the final value is approached. To improve this aspect, modifications of the gradient method have been made to include the second-order terms. These methods are known as methods based on second variations. Kelly [8], Breakwell, Speyer, and Bryson [9], Kelly, Kopp, and Moyer [10], and Merriam [11] have contributed to the development of this second-order method. Although the second-order method does accelerate convergence of the first-order method, it requires good initial estimates of the initial control and trajectory. Methods that have characteristics of both the first-order and second-order methods have been developed such as the conjugate gradient method by Flectcher-Reeves [12], and Powell [13], and the variable metric method by Davidon-Fletcher-Powell [14]. Conjugate gradient methods are much better than first-order methods and approximate the performance of the second-order methods. A final alternative indirect method is the variational metric method which is usually good but is not always practical to implement.

Formulation of Optimization Problem

The system state equations 1, 2, 3 and 4 are a set of four first order, non-linear equations. The control inputs to the system are the current or voltage applied to the electrolhydraulic valve.

Contraints

Inequality constraints

Controls:

$$0.0 < U_1(t) < U_m$$

$$0.0 < U_2(t) < U_m$$

States:

$$0.0 < X_3(t)$$

$$X_4(t) < X4M$$

Initial conditions

$$X_1(0) = X10$$

$$X_2(0) = 0.0$$

$$X_3(0) = X_4(0)$$

Desired conditions

$$X_1(t_f) = X1D$$

$$X_2(t_f) = 0.0$$

Performance measure

$$J(X,U) = h(X(t_f)) + \int_0^{t_f} g(X,U)dt$$

where

$$h(X(t_f)) = (R_1/2)(X1D - X_1(t_f))^2$$
$$+ (R_2/2)(X2D - X_2(t_f))^2$$
$$+ (R_3/2)(X3D - X_3(t_f))^2 + (R_4/2)(X4D - X_4(t_f))^2$$

$$g(X,U) = RU_1\sqrt{P_2 - X_3} \quad \text{for } U_1 > 0$$

Note

R, R_1 to R_4 are weighting factors

X1D to X4D are desired states

t_f is the final time

The Variational Method

The method of calculus of variations used in many control applications will be considered. The Hamiltonian corresponding to the set of nonlinear equations and the performance measure and the constraints will be minimized. This method introduces new parameters called costates are obtained by integrating backward from final time to initial time. Problems of this type are called Two-Point Boundary Value Problems because of the split boundary conditions.

First the problem without inequality constraints will be considered. The Hamiltonian to be minimized for this particular problem will take the form:

$$H(X,p,U) = g(X,U) + p^T f(X,U)$$

where p is a vector of costates.

The costates are found from the necessary conditions for optimization which include [15]

$$\dot{p} = -\frac{\partial H}{\partial x}, \frac{\partial H}{\partial u} = 0$$

with the final conditions

$$p_1(t_f) = -R_1(X1D - X_1(t_f))$$

$$p_2(t_f) = -R_2(X2D - X_2(t_f))$$

$$p_3(t_f) = -R_3(X3D - X_3(t_f))$$

$$p_4(t_f) = -R_4(X4D - X_4(t_f))$$

Inequality Constraints

The penalty function approach will be used to incorporate inequality constraints on states and controls into the previously defined optimization problem.

Let $SC_i(X)$ and $CC_i(U)$ be constraint functions

required to satisfy

$$SC_i(X) < 0, \; i = 1,2$$

$$CC_i(U) < 0, \; i = 1, \ldots 4$$

The state constraints applied here are to prevent cavitation and excessive pressure

$$SC_1(X) = -X_3(t)$$

$$SC_2(X) = X_4(t) - X4M$$

The constraints on control limit the range of control values

$$CC_1(u) = -U_1(t)$$

$$CC_2(u) = U_1 - U_m$$

$$CC_3(u) = -U_2(t)$$

$$CC_4(u) = U_2 - U_m$$

The approximate inequality constraints on state and controls are incorporated into a new Hamiltonian as

$$H'(X,p,U) = H(X,p,U) - \sum_{i=1}^{2} K_i h(SC_i)[SC_i(X)]^2 + \sum_{i=1}^{4} \mu_i \, CC_i(U)$$

where K_i = positive penalty weight number .....i=1,2

$h(SC_i)$ = 0 .....$SC_i(X) < 0$

= 1 .....$SC_i(X) > 0$

μ = weighting function on constraint U_i

Numerical Solution of Optimal Control Problem

The Fletcher [16] conjugate gradient search technique was used after the failure of the gradient method to converge rapidly enough. Details can be found in [2]. Convergence to a local minimum generally occured in 10 or 12 iterations and always less than 15 iterations requiring about 250 cpu seconds execution time on the Cyber 74 computer. About 80% of that time was consumed by integrating state equations forward and integrating the costate equations backwards. The amount of time is strongly dependent on the stopping criteria which is the incremental improvement in performance achieved.

The local minimum found is dependent on the nominal solution used as a guess. A bang-bang control was used as an input with u_1 (the four way valve) opened to maximum initially. At some switching time u_1 was made zero and u_2 made maximum to allow recirculation. The switching time was the primary variable in the different nominal controls considered. The smallest local minimum was assumed to be a global minimum. The control resulting from the optimization was a bang-singular-bang control. Depending on the relative weighting given to the penalty terms the control profile varied somewhat. For a high penalty on final position error (R_1 = 20,000) and lower penalties on energy (R = 100) and velocity error (R_2 = 20) a control profile as shown in Figure 4 resulted. The 5 inch move was made in .2 seconds. The resultant states over time from simulation are shown in Figure 5. The energy savings over the conventional 4-way valve control is 28%. Changing the weighting on energy by a factor of four resulted in only a 1% change in energy consumed.

Reducing the final time for the motion from 200 milliseconds to 180 milliseconds requires 11% more energy. If the minimum time is allowed for the move the conventinal four way valve and the controlled fluid recirculation will result in the same energy requirements. Increasing the final time from 200 milliseconds to 250 ms results in 15% less energy consumption, according to simulations.

Recall that the model time delay (about 10 ms) was not incorporated into the otpimal control analysis. It is important to realize that the resulting optimal control is also optimal for the system with time delay. The effects of the control will simply be delayed by 10 ms.

Modification of Control for Implementation

For purposes of implementing the control on the breadboard system certain simplifications were made. The resulting history shown in Figure 6 consists of constant control levels. U_1 (the 4-way valve) is open to the maximum during acceleration of the load while U_2 (the 2-way valve) is closed. At about 0.07 seconds the control is switched. U_2 is opened fully to allow recirculation and U_1 is reduced to an intermediate level corresponding approximately to the singular are of the exact solution. The energy term in the performance integral with the modified control is within 0.1% of the optimal control for the simulated motion. The other terms of interest in the performance integral are shown in Table 2.

The Experimental Results

The modified control was implemented on a TI 990/4 microcomputer to control the breadboard system. In addition to the optimal control law determined by a switching point based on position, a proportional feedback control law was used after the final position was reached. This prevented drift away from the position due to the inexact null position of the valve. Only the four-way valve was used for the proportional control.

TABLE 2
Comparison of Numerical to Approximate Results

	Conjugate Gradient Method	Approximate Result
X1f	2.4989	2.5001
X2f	.128	.174
X3f	557.07	568.61
X4f	679.84	629.00
tf	200 mS	200 mS
Energy Terms	4.84205	4.8475
State Error	.3276	.6055

The computer output went to a 12-bit digital to analog converter, then to amplifiers, then to the two valves. Position was measured by a linear variable differential transformer (LVDT), converted to a 12 bit digital signal and used to determine the switching point. The measurements were also recorded and plotted.

The modified control history and the mass position are shown in Figure 7. The successful completion of the desired move was observed in all cases. The analysis did not incorporate the 10 ms delay known from previous experiments to exist. Accounting for this delay the agreement between simulation and experiment were quite good.

Summary and Comments

A hydraulic circuit and optimal control law for controlled fluid recirculation have been devised to reduce energy consumption from that of a conventional four-way servo valve. The simulations for moving a 100 lb. mass horizontally 5 inches in 0.210 sec. show an energy savings of 28%. The control history was simplified and implemented on a breadboard system and the successful operation verified experimentally.

The optimization penalized energy consumed and the final position and velocity errors. The controlled fluid recirculation concept could also be used to follow a specified trajectory. This problem could be simpler to implement than the one considered. Other types of loads which store energy in potential and/or kinetic form could also benefit from this technique.

Further study should consider the sizing of the two valves and the use of a less expensive on-off two way valve. Further attention should also be given to the possibility that the global optimum has not been found. Multiple switching points may exist for movements greater than a certain distance. The results would be a pulsed operation alternating between accelerating and coasting.

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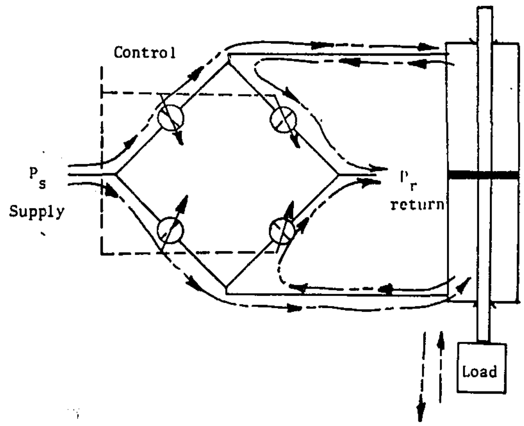

Fig. 1. The Four-way valve as a Wheatstone bridge.

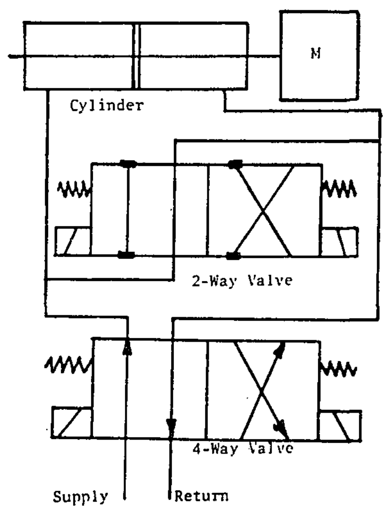

Fig. 2. The Experimental Circuit as Constructed.

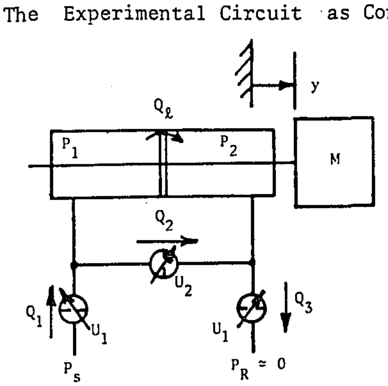

Fig. 3. Schematic with Notations for Motion in One Direction.

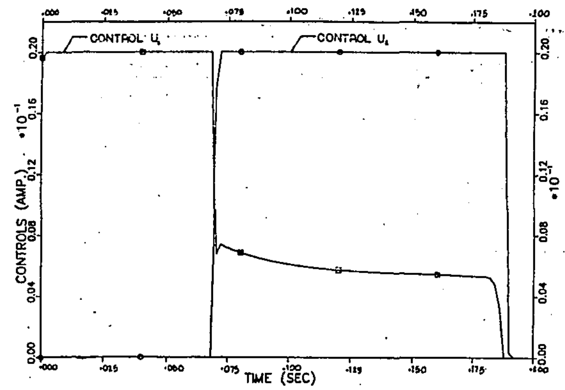

Fig. 4. Optimal Control Result for R = 100., R_1 = 20,000, R_2 = 20. (Conjugate Gradient Method).

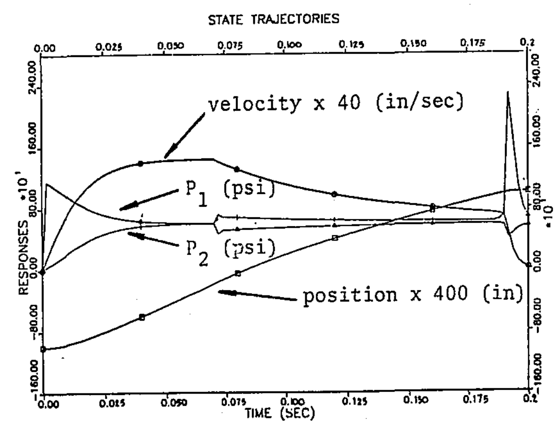

Fig. 5. Simulation of Controls in Figure 4.

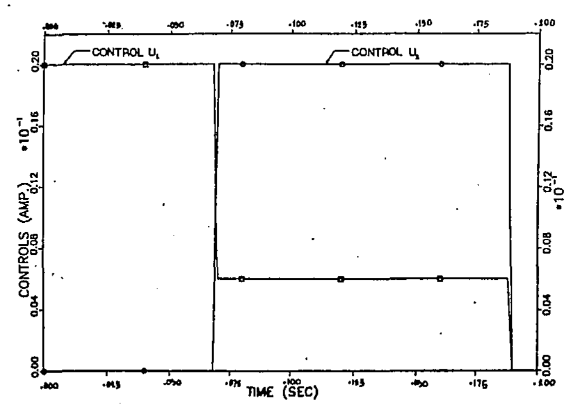

Fig. 6. Approximation of Optimal Control for R = 100, R_1 = 20,000, R2 = 20.

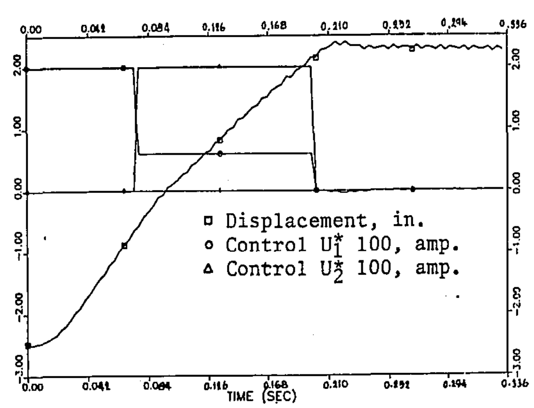

Fig. 7. Plots of Displacement and Control History from Experiment.